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THE COMPUTATION OF TRUSS DEFLECTIONS BY THE METHOD OF ELASTIC WEIGHTS

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THE COMPUTATION OF TRUSS DEFLECTIONS BY THE METHOD OF ELASTIC WEIGHTS

(AIRPLANE BRANCH REPORT)

▽

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THE COMPUTATION OF TRUSS DEFLECTIONS BY THE METHOD OF ELASTIC WEIGHTS

INTRODUCTION

1. In chapter 2 of Airplane Design, two methods of computing the deflections of trusses are suggested—those of the Williot diagram and the method of work. Only the latter is described in detail. Both of these methods are extremely tedious when it is desired to compute the deflections of all the points on one chord of a long shallow truss, such as a truss-type wing spar. The Structures Unit has for a long time believed that a simpler and quicker method of computing truss deflections would be a great help to the designer, and its members have made several unsuccessful attempts to develop one. The desirability of some such method has become much greater in the recent past with the development of metal wing spars, many of which are of truss types. Recently, the writer found in Parcel and Maney's book, Statically Indeterminate Structures, an outline of the method of elastic weights, which indicated to him that this method would fill the requirements of the aeronautical engineer in a much more satisfactory manner than those mentioned above. The description of the method given by Parcel and Maney was quite sketchy, and the conventions for signs, etc., differed from those usually employed in aeronautical work in this country, but the basic principles were clearly stated. This report is intended to explain the underlying principles, and by the use of numerical examples to illustrate the application of the method with such clarity that it can be employed by the airplane designer in cases where it will be useful.

2. The fundamental principles of truss computation will be discussed first, and the theoretical basis of the method of elastic weights briefly explained. The application of this method will then be shown, first for simple and then for more complex cases. In doing this the derivation of the special rules for the given case will first be described and their application illustrated by a numerical example. The appendix gives the rules and definitions of special terms in the most general form for the benefit of the designer who wishes to refer to the report to guide him in a specific computation.

3. The fundamental principle for the computation of the deflections of a truss composed of members subjected to axial loads only is as follows: The deflection of any point a of the structure due to the change in length of any member R is numerically equal to the change in length of member R multiplied by the load that would be produced in member R by a load of unity acting at a in the direction in which the deflection is desired. Expressed algebraically

$$\delta ar = \Delta L_r . s_r$$

where δ_{ar} is the deflection at a due to change of length of member R. ΔL_r is the change in length of member R and s_r is the load that would be caused in member R by a load of unity at a acting in the direction in which the deflection is to be measured. As this is the fundamental formula for truss deflections, and its derivation is given in all textbooks that cover this subject, it will be assumed that the reader is familiar with it.

4. The usual method of applying this formula is that illustrated in article 50 of Airplane Design. The disadvantage of this procedure is that a long and tedious computation must be made for each point at which the deflection is desired. By using the device of elastic weights, the deflection of all points along the truss can be obtained with little more competition than is needed for a single point by the method of work.

5. For a given loading the value of ΔL_r, or $P_r L_r / A_r E_r$ as it is more often expressed, is a constant. The value of s_r, however, depends on the location of the unit load. If a curve be drawn with each ordinate equal to the value of s_r when the dummy unit load is acting on the main chord [1] at the location represented by the corresponding abcissa, we will have what is called the influence line for stress in member R. A little thought should show that this line is also to some scale, as yet, undertermined, the curve representing the deflection of the main chord of the truss due to the change in length of member R. It would be possible to draw such a curve for each member of the truss, and by algebraic addition of ordinates obtain the elastic curve of the truss under the given load. This would be very tedious, however, and is not often done.

6. In the normal types of trusses, it will be found that the influence lines for load in the different members are geometrically similar to the bending moment curves for certain simple loads. If, then, for each member the proper load is computed and applied to a phantom beam paralleling the actual truss, and the curve of bending moments on the phantom beam due to these loads is calculated, this bending moment curve will be, to some scale, the elastic curve of the truss, and the deflections of the truss can be determined from it. The method takes its name from the fact that these hypothetical loads are called "elastic weights" or "elastic loads." The methods used to compute the elastic loads for the different types of truss can be described and explained most simply by the aid of numerical examples.

7. In these numerical examples the conventions regarding the signs of forces, shears, moments, slopes, and deflections will be those used in Airplane Design, volume 1, and listed in the last paragraph of page 289 of that book.

8. As the trusses dealt with in airplane design usually have chords that are horizontal or nearly so, and it is

[1] On this report the truss chord, the deflections of which are desired, is called the main chord. See paragraph 4 of the appendix.

INDEX

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the vertical deflections of the joints of a single chord that is desired, the numerical examples below will be confined to such cases. Anyone who fully understands these cases should have no difficulty in applying the method to determine the deflections in other directions, or of other types of trusses in case they are desired.

9. Whenever the discussion is limited to a single member, subscripts will be omitted. Then, if the computation of the elastic weight of a member called *bd*, its length will be designated as L instead of L_{bd}, and its change in length due to load as ΔL instead of ΔL_{bd}. If the discussion involves more than one truss member, the proper subscripts will be used.

SIMPLY SUPPORTED TRUSS SPANS

10. The first example will be a simply supported parallel chord truss span like that shown in Figure 1. This truss has three types of members, chord members, web members, and end posts. Each of these types will be considered separately, the method of computing their elastic weights and the basis of the computation being described in detail. As the deflections sought will be those of the lower chord joints, its lower will be considered the "Main chord" of the truss.

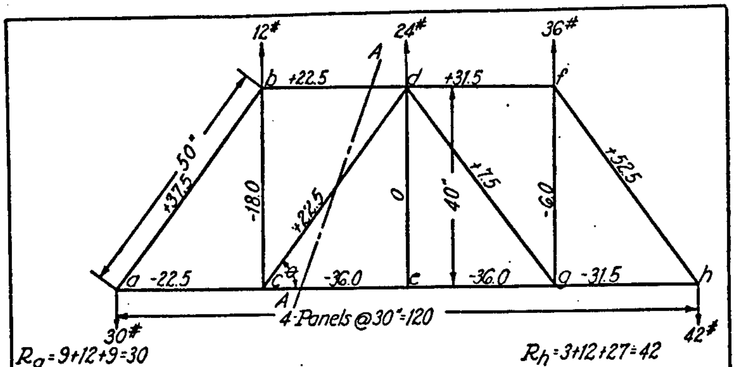

Fig.1
Truss Diagram

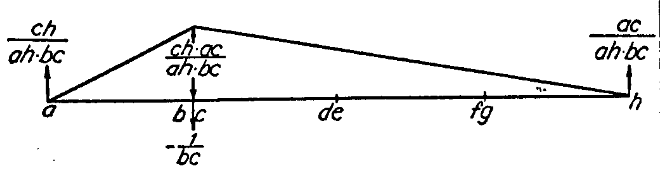

Fig.2
Influence line for load in "bd"

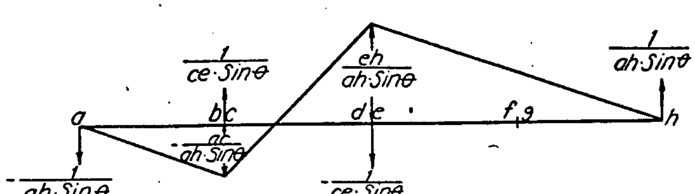

Fig.3
Influence line for load in "cd"

CHORD MEMBERS

11. Consider the chord member *bd*. As the load in this member can be determined by the method of sections, taking moments of the forces on that portion of the trusses on either side of section A–A, about joint *c*, joint *c* can be designated the moment center [2] and section A–A the index section of member *bd*. The influence line for load in *bd* due to a unit vertical load moving along the main, in this case the lower chord of the truss will be a triangle with its apex at the moment center *c*, as shown in Figure 2. The maximum ordinate of this influence line will be at *c*, and its magnitude will be equal to $+\frac{ch}{ah}+\frac{ac}{bc}$, the unit load being assumed to act upward when positive, and the plus sign to indicate tension. But this is numerically equal to the bending moment at *c* on a "phantom beam" supported at *a* and *h*, due to a downward load of $1/bc$ acting at *c*. Therefore the moment curve for a load of $-1/bc$ acting at *c* on the phantom beam may be considered as the influence line for load in *bd*, or, using the nomenclature of paragraph 3 above, as the influence line for *s*.

12. The change in length of *bd*, which we will call ΔL, is equal to PL/AE. This quantity is a constant for any given loading. Since the product of ΔL and *s* for any point gives the deflection of that point under the given loading, the ordinates of the curve in Figure 2 multiplied by ΔL will give a curve of the deflection of the truss due to the change of length of member *bd*, or the same result can be obtained by adjusting the scale of Figure 2. Another way of looking at the matter is to say that the curve of moment on the phantom beam due to a load of $-\Delta L/bc$ is the deflection curve of the truss due to the deformation under load of member *bd*. This load is called the "elastic weight" of member *bd*.

13. A similar analysis could be made for each of the other chord members of the truss, from which the following generalizations could be derived. The elastic weight of any chord member of a simply supported truss is numerically equal to $\Delta L = PL/AE$, the change in length of that member, divided by the perpendicular distance from that member to its moment center. The curve of bending moment on a phantom beam supported at the same points as the actual truss span, due to a load equal to the elastic weight of a chord member acting on the phantom beam at the moment

[2] On this report the point about which moments would be taken to obtain the load in a truss member by the "Method of moments" is called the moment center of that member. The section through the truss used to determine the load in a member by either the "Method of moments" or the "Method of shears" is called the "index section" of the member. (See also par. 8 of the appendix.)

center of the member in question, will be a curve of the deflection of the truss due to the change in length of the member in question. The curve of bending moments on the phantom beam due to the elastic weights of more than one chord member acting simultaneously, each at the moment center of the member in question, will be a curve of the deflection of the truss due to the changes in length of that group of chord members.

14. The moment centers of the upper chord members will be joints on the lower chord, while those of the lower chord members will be joints on the upper chord. As the elastic weights all act in a vertical direction, the vertical locations of their points of application have no influence on the curve of bending moment on the phantom beam, so that the elastic weight of member *bd*, the moment center of which is point *c*, and that of member *ac*, the moment center of which is point *b*, will be applied to the phantom beam at the same point, *b* being vertically above *c*.

15. Table 1 gives the computations of the elastic weights for the truss shown in Figure 1 when subjected to the loads indicated. The first six lines give the computations of the elastic weights of the chord members. In this case, as the chords are parallel, the moment arms of the chord members are all equal to the truss depth, *h*, and this fact is taken advantage of in the tabulation of computations. The last two columns of the table give the signs of the elastic weights, indicating whether they are to be assumed as acting up or down on the phantom beam, and the points of application of the elastic weights to the phantom beam, in the case of these chord members, their respective moment centers.

16. The signs of the elastic weights of the chord members are determined as follows: For lower chord members, the sign is plus if the member is in tension and minus if it is in compression. For the upper chord, the sign is minus if the member is in tension and plus if it is in compression. These rules can be generalized by saying that "The sign of the elastic weight of any chord member of a simply supported truss is that of the bending moment on the truss indicated by the type of the load in that member, whether tension or compression." In this way, the familiar rules for determining the sign of the bending moment on a beam or truss can be used to determine the sign of elastic weights.

TABLE 1.—*Computations of elastic weights for truss of Figure 1*

Member	P	L	A	PL/A	h	p	$\sin\theta$	PL/Ah	$PL/Ap\sin\theta$	Sign	P. A.
bd	+22.5	30	1.25	540	40	------	------	13.50	------	−	c
df	+31.5	30	1.50	630	40	------	------	15.75	------	−	g
ac	+22.5	30	1.50	450	40	------	------	11.25	------	−	b
ce	−36.0	30	1.80	600	40	------	------	15.00	------	−	d
eg	−36.0	30	1.50	720	40	------	------	18.00	------	−	d
gh	−31.5	30	1.75	540	40	------	------	13.50	------	−	f
ab	+37.5	50	2.50	750	------	30	0.8	------	31.25	↷	ac
bc	−18.0	40	1.50	480	------	30	1.0	------	16.00	↷	ac
cd	+22.5	50	1.25	900	------	30	.8	------	37.50	↷	ce
de	0	40	1.00	0	------	30	1.0	------	0	−	------
dg	+7.5	50	1.25	300	------	30	.8	------	12.50	↶	eg
gf	−6.0	40	1.00	240	------	30	1.0	------	8.00	↶	gh
fh	+52.5	50	3.50	750	------	30	.8	------	31.25	↶	gh

P=axial load in member due to external loading shown, in pounds.
L=length of member in inches.
A=sectional area of member in square inches.
h=depth of truss in inches.
p=length of truss panel in inches.
θ=angle between web member of truss and the horizontal.
The signs indicate the directions in which the elastic weights act, + indicating an upward force, and − a downward one. Where the elastic weights are moments, the sign shows whether they are clockwise or counterclockwise.
The column headed P. A. shows the points of application of the elastic weights.
The modulus of elasticity, E, is omitted from the above computations as it is assumed to be the same for all members. To obtain the true deflection it would be necessary to divide the result of the computations from the elastic weights shown in the above table by the proper value of E.

17. In a parallel chord truss there are no moment centers for the web members and it is obvious that the methods of obtaining the elastic weights of chord members can not be applied directly to the web members. It is possible, however, to use the same fundamental method and find a loading for the phantom beam that will have a bending moment curve of the same shape as the influence line for load in the web member in question. In the case of a web member of a parallel chord truss, this loading consists of two equal and opposite loads applied at the points where the influence line for load in the web member changes direction.

18. Consider member *cd* in Figure 1. The influence line for load in that member will be that shown in Figure 3. As can be seen from the figure, this is equivalent to the moment curve for an upward force of $1/(ce \times \sin\theta)$ at *c* and an equal but opposite force acting at *e*, where θ is the angle between the web member in question and the horizontal. From this it can be seen that the curve of deflection of the truss due to change in length, ΔL, of member *cd* will be the moment curve of the phantom beam supported at *a* and *h* under loads of $\Delta L/(ce \times \sin\theta)$ acting up at *c* and down at *e*. This pair of loads is the elastic weight of the member *cd*.

19. It is to be noted that these forces are applied at the ends of the lower chord member cut by the index section for member *cd*, and that each is equal to ΔL of that web member divided by the distance to it from the far end of the lower chord member. If we call the far end of this lower chord member the secondary moment center of the web member, the following rule can be formulated: "The elastic weight of a web member of a parallel chord truss consists of two loads of equal magnitude but acting in opposite directions, one at the

secondary moment center of the web member, and the other at its connection with the lower chord. The magnitude of each of these forces will be equal to the change in length, ΔL, of the web member divided by the distance to that member from its secondary moment center." This is the "Web member rule" of paragraph 9 of the appendix and will be referred to by that name in the remainder of this report.

20. The web member rule for the magnitude of the elastic weight, in the form stated above, applies only to computations of the deflection of the lower chord of the truss. If the deflection of the upper chord is being computed, the rule would be changed by substituting the word "upper" for "lower" in the rule and also in the definition of the secondary moment center. In the rules and definitions in the appendix to this report, this has been taken care of by referring to the "main chord" instead of using the terms "upper" and "lower."

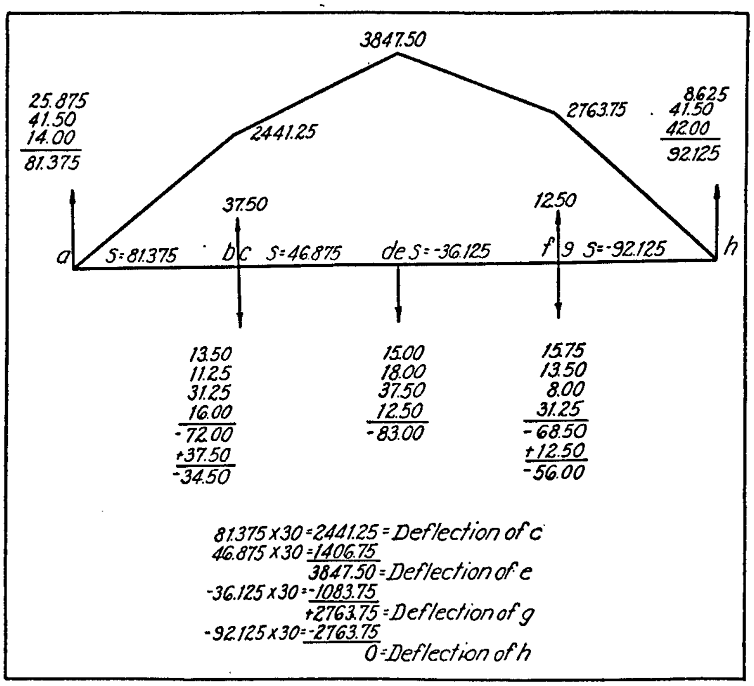

FIG. 4.—Computation of truss deflection

21. The rule for the sense of the moment constituting the elastic weight of a web member is that when the shear on a section cutting the truss through a member is positive, the elastic weight of that member is a counterclockwise moment, while if the shear is negative, the moment of the elastic weight is clockwise.

22. Table 1 gives the computation of the elastic weights of the web members of the truss of Figure 1. In this tabulation advantage is taken of the fact that all bays of the trusses are of the same length, $p=30$ inches, and the slopes of the web members are either 90° or the angle whose sine is 0.8.

END POSTS

23. End posts can be considered as either web or chord members. Consider the case of member *ab*. This member has a moment center at *c*. Its elastic weight according to the rule for chord members will be a single downward or negative load acting at point *c*. This elastic weight will be negative, as *ab* is an upper chord member in tension under the loading shown in Figure 1, and a beam with the upper fibers in tension is subjected to a negative moment. The magnitude of the elastic weight acting at *c* will be equal to ΔL, the change in length of *ab* divided by the perpendicular distance from point *c* to the line *ab*. This distance is equal to $ac \times \sin\theta$ where θ is the angle between *ab* and the horizontal. The elastic weight of *ab* is therefore $\Delta L/(ac \times \sin\theta)$ acting down at *c*.

24. Suppose that *ab* be treated as a web member. As the shear on any section of the beam through *ab* is negative, the elastic weight will consist of an up load at *a* and a down load at *c*, *c* being the secondary as well as the true moment center of the member. As *a* is one of the supports of both the actual truss and the phantom beam, the load acting at that point may be neglected, the down load at *c* thus becoming the elastic weight of the member. The magnitude of this down load as derived from the rule for web members is $\Delta L/(ac \times \sin\theta)$ the same as according to the rule for chord members.

25. The computations of the elastic weights of the end posts of the truss of Figure 1 are given in Table 1. In that place they are treated as web members, but the only differences that would appear in the computations if they had been treated as chord members are that the signs of the elastic weights of *ab* and *fh* would have been minus signs instead of the notations for clockwise and counterclockwise moments, respectively, and the elastic weights of the two end posts would have had only one point of application listed instead of two. In this case it is more convenient to treat the end posts as web members as the distance to each from its moment center is the same as that of the sloping web members from their secondary moment centers while the distances to the other chord members from their moment centers is a different figure. By treating the end posts as web members, the tabulation of the computation is made more uniform and logical.

COMPUTATION OF DEFLECTIONS

26. Figure 4 is a diagram of the phantom beam corresponding to the truss of Figure 1, showing the elastic weights computed in Table 1 acting at the proper points of application. Positive or upward acting loads are shown by arrows above the line representing the phantom beam, and negative or downward loads by arrows below that line. The computations of the deflections of the truss are all shown on the figure. The steps are as follows:

a. Computation of net elastic loads at the various points of load application on the phantom beam other than the points of support.

b. Computation of the reactions at the points of support of the phantom beam. In this computation advantage was taken of the fact that all of the loads were applied at either the mid-point or the quarter points. The mid-point load was divided equally between the supports, and the quarter-point loads three-quarters to the near and one-quarter to the far support. The resulting partial reactions were then added to obtain the total reactions.

c. Computation of the shear in each section of the phantom beam. The resulting value is indicated on the appropriate section of the beam in the figure.

d. Computation of the moments on the phantom beam at the points corresponding to the lower chord joints of the original truss, the deflections of which are desired.

27. In making the computations of Table 1 and Figure 4, the modulus of elasticity, E, was omitted as it was assumed to be the same for all members. The final results obtained therefore are not the deflections of the truss, but these deflections multiplied by E. This practice was followed as it resulted in the use of numbers of reasonable size instead of very small decimals, making the computations easier to carry out, and entailed no loss of clarity. In any practical computations of trusses in which the materials are all of the same material it will usually be advisable to follow the same practice. When different materials are used, one of them should be considered as standard and the computations made, using not the absolute values of E in pounds per square inch, but the values in terms of E of the standard material. As the last step the bending moments on the phantom beam can be divided by the E of the standard material to obtain the deflections of the actual truss.

28. It is to be noted that the deflections obtained in the above computations are those of the lower chord joints. The computation of the deflections of the upper chord joints would be the same except that the points of application of the elastic weights of members *bc* and *fg* would have been *b* and *d* instead of *a* and *c* for the first, and *d* and *f* instead of *g* and *h* for the second. That the weights of these two members would be the only ones to be changed is shown by the fact that the deflection of *b* will differ from that of *c* only by the change in length of *bc*, while a similar relation exists between *f* and *g*. In this case the deflections of *d* and *e* will be the same, as they are connected by member *de* which is not subjected to load.

CHECK BY METHOD OF WORK

29. In order to prove the validity of the computation of deflections as made above by the method of elastic weights, the deflections of the same points have been computed by the method of work. The computations are given in Table 2. The values of PL/A are taken from Table 1. The quantities u_c, u_e, and u_g, are the loads in the various members due to unit loads at *c*, *e*, and *g*, respectively. The value of PL/A, which is ΔL, for each member is multiplied by each of the stresses due to unit load. The columns of values of PLu/A are summed up to obtain the deflections of the points where the unit loads were assumed to have been applied. The agreement between the deflections obtained by the elastic weight method and that of work is absolute, though there would have been a slight difference due to dropping of significant figures after the third or fourth, had not the necessity of this been obviated by careful choice of loads and dimensions for this example.

TABLE 2.—*Check computation by the method of work*

Member	PL/A	u_c	PLu_c/A	u_e	PLu_e/A	u_g	PLu_g/A
bd	+540	+0.5625	303.750	+0.375	202.50	+0.1875	101.250
df	+630	+0.1875	118.125	+0.375	236.25	+0.5625	354.375
ac	−450	−0.5625	253.125	−0.375	168.75	−0.1875	84.375
ce	−600	−0.3750	225.000	−0.750	450.00	−0.3750	225.000
eg	−720	−0.3750	270.000	−0.750	540.00	−0.3750	270.000
gh	−540	−0.1875	101.250	−0.375	202.50	−0.5625	303.750
ab	+750	+0.9375	703.125	+0.625	468.75	+0.3125	234.375
bc	−480	−0.7500	360.000	−0.500	240.00	−0.2500	120.000
cd	+900	−0.3125	−281.250	+0.625	562.50	+0.3125	281.250
de	0	0	0	−1.000	0	0	0
dg	+300	+0.3125	93.750	+0.625	187.50	−0.3125	−93.750
fg	−240	−0.2500	60.000	−0.500	120.00	−0.7500	180.000
fh	+750	+0.3125	234.375	+0.625	468.75	+0.9375	703.125
			2,441.250		3,847.50		2,763.750

SPECIAL CASES OF SIMPLY SUPPORTED TRUSSES

30. In the numerical example given above, the truss is of the simplest type, with parallel chords and equal panels. The designer, however, will often wish to compute deflections of more complex trusses, and for the method of elastic weights to be made generally applicable, the modifications of the rules explained above must be described. The cases that will be taken up here will be as follows:

a. Vertical end posts.

b. The Warren truss, with parallel chords and equal bays.

c. Trusses with unequal bays.

d. Trusses with nonparallel chords.

e. Secondary web members.

VERTICAL END POSTS

31. Vertical end posts should cause no difficulty, being treated as either chord or web members in the same manner as sloping end posts. As in the numerical example above, it will usually be found most convenient to treat them as web members, particularly when the truss is of equal panels.

THE WARREN TRUSS

32. The simple Warren truss should offer little difficulty. The moment centers of the chord members are well defined, those for the upper chord members being at the joints of the lower chord, and those of the lower chord members at the joints of the upper. The points of application of the elastic weights of the web members are the joints at the ends of the member of the main chord cut by the index section of the web member.

33. If the deflection of the lower chord is being obtained and the elastic weights of the lower chord

members of the truss are applied to the phantom beam under the upper chord joints which are their moment centers, the moment curve of the phantom beam will not be a true deflection curve of the lower chord of the truss. The deflections indicated for the lower chord joints will be correct, but the lines of the moment curve between the adjacent lower chord joints will not be straight like the members of the truss, but will change direction at the points opposite the upper chord joints. The difference will usually be small, and in any event should cause no difficulty as the true deflection curve of the lower chord can be obtained in either of the two ways if it is needed for any reason. The simplest way is to neglect the points representing the moments on the phantom beam at the points representing upper chord joints, drawing straight lines between the points representing the moments at the lower chord joints. The other method is to divide elastic weights at the upper chord joints between the nearest lower chord joints in inverse proportion to the distances to those joints.

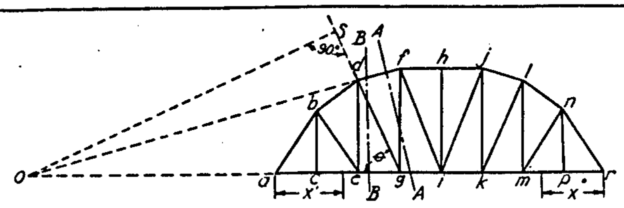

Fig.5
Diagram of truss

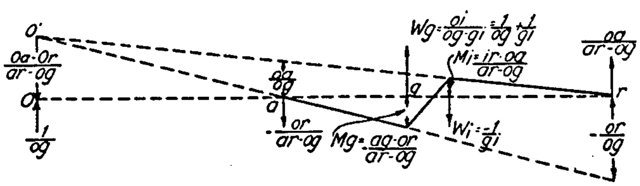

Fig.6
Influence line for load in gf

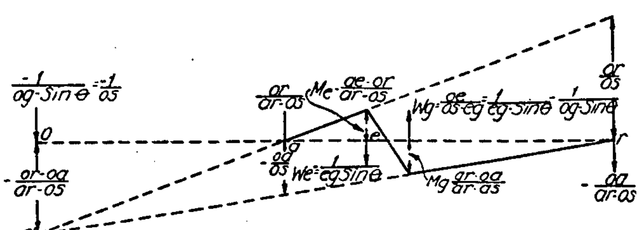

Fig.7
Influence line for load in dg

34. In computing the value of sin θ the numerical value is all that is needed, the sense of the moment caused by the elastic weight of the web member being determined by the shear on the beam at the section in question. Thus although adjacent web members are subjected one to tension and the other to compression, if the shear is of the same sign at the sections through both members the moments of the elastic weights of these members will both have the same sense. This is illustrated by members bc and cd in Table 1.

TRUSSES WITH UNEQUAL BAYS

35. Where trusses have unequal bays there is no essential change in the method of application of elastic weights. The only difference in the computations is that the slope θ of the diagonal web members and the panel length by which the ΔL values of the web members are divided are no longer constants. This makes the computations a little more tedious and usually makes it advisable to have one or two additional columns in the computations of the elastic weights if they are tabulated. This variation from the simple case should therefore give the designer no special difficulty.

NONPARALLEL CHORDS

36. The chord members of trusses with nonparallel chords offer no special difficulties, the elastic weights being applied at the moment centers of the various members and equal to the changes in length divided by the perpendicular distances from the members to their moment centers. In the case of sloping chord members, however, the perpendicular distances will not be equal to the truss depth but will be equal to the horizontal distance from the moment center to the chord member in question multiplied by the sine of the angle between the member and the horizontal, or what amounts to the same thing, the vertical distance multiplied by the cosine of the angle.

37. In the cases of web members, the computation of the elastic weights is considerably modified.

Consider the truss shown in Figure 5 and follow through the computation of the elastic weight of member gf. The load in member gf can be obtained by the method of moments, and this will be done to obtain the influence line for that member. The section taken will be A–A cutting members df, gf, and gi, and the moment center will be O, the intersection of df and gi produced.

38. When the unit load is on the lower chord between i and r, the load in gf will be $s=\frac{x}{ar}\cdot\frac{Oa}{Og}$ where x is the distance from r to the unit load. The load in gf obtained from this equation is plotted in Figure 6 as line $O'r$, the ordinates at O, a, and i being shown on the figure.

39. When the unit load is between a and g, the value of s is given by the equation $s=\frac{-x'}{ar}\cdot\frac{Or}{Og}$ where x' is the distance from the unit load to a. This expression for s is plotted in Figure 6 as line $O'a$, the ordinates at O, g, and r being shown.

40. The influence line for s is then the solid line of Figure 6, and it is desired to determine the load for which this line is the curve of bending moments on the phantom beam supported at a and r. It is obvious that this load will consist of forces at g and i of unequal magnitude and acting in opposite directions.

41. The first step in finding these forces is to determine the reactions at a and r, which can be done very

easily by dividing the moments at g and i by the distances ag and ir, respectively. These reactions are then $\frac{-Or}{ar.Og}$ at a, and $\frac{Oa}{ar.Og}$ at r.

The load at i can then be found by taking moments about g.

$$W_i=\frac{-1}{gi}\left(\frac{gr.Oa}{ar.Og}+\frac{ag.Or}{ar.Og}\right)=\frac{-1}{gi}\left(\frac{gr.Oa+ag.Oa+ag.ar}{ar.Og}\right)=\frac{-1}{gi}\left(\frac{ar.Oa+ar.ag}{ar.Og}\right)=\frac{-1}{gi}\text{ since }Oa+ag=Og.$$

The load at g can be obtained by applying $\Sigma V=O$.

$$W_g=\frac{1}{gi}+\frac{Or}{ar.Og}-\frac{Oa}{ar.og}=\frac{1}{gi}+\frac{1}{og}=\frac{Og+gi}{og.gi}=\frac{Oi}{og.gi}$$

42. The values just obtained for W_g and W_i when multiplied by ΔL, together constitute the elastic weight of member gf. It will be simpler for some purposes, however, to consider the elastic weight to be their resultant, so as to consider it a single force instead of a pair of forces. This resultant force will be equal to the algebraic sum of the forces, which is evidently $\Delta L/Og$. Its location can be found by taking moments about g, letting x be the distance from g to the point of application of the resultant. Then

$$x\,\frac{\Delta L}{O_g}=-gi\left(\frac{-\Delta L}{gi}\right)$$

whence $x=Og$. Therefore the resultant elastic weight acts at point O, the moment center of the web member in question.

43. As Og is the moment arm of member gf about its moment center, it will be seen that in this case the elastic weight of a web member can be obtained by use of the rule employed for the determination of the elastic weight of a chord member. The principal difference is that instead of applying this elastic weight to the phantom beam as a single load at the moment center, it is applied as a pair of loads at the ends of the lower chord member cut by the section used in obtaining the load in the web member, the resultant of these loads being the single force at the moment center.

44. If the chords had been parallel, Oi and Og in the expression for W_g would both become infinite, and their ratio equal to unity. In such a case the expression for the elastic weight of the web member would become identical with that given above for vertical web members when parallel chord trusses were under discussion.

45. Before taking up the question of the rules for the direction of the elastic weights of web members, the computation of that property for a sloping web member will be considered briefly, using member dg of Figure 5 as an example.

46. The influence line for load in dg is shown in Figure 7, it having been obtained in the same manner as the influence line in Figure 6 for member gf. The chief differences in the computation are that the index section is BB cutting members df, dg, and eg instead of members df, gf, and gi. Also, as the member dg has a slope θ, its moment arm about its moment center is the sloping line Os which is equal to $Og.\sin\theta$. It is not believed that the computations of the ordinates of the influence line-moment curve, of the elastic weight of member dg, the reactions at a and r, and the resultant at O will cause difficulty.

47. From Figures 6 and 7, it will be seen that in the case of members of nonparallel chord trusses, the magnitudes of their elastic weights can be obtained by using the rule for chord members of trusses—namely, by dividing the respective values of ΔL by the distances to the members from their moment centers, these loads being assumed to act at the moment centers in question. When the moment center is at infinity, the elastic weight becomes a couple instead of a single force. The elastic weight is applied to the phantom beam at the moment center only when that moment center is one end of a member cut by the section used in determining the loads in the member in question—i. e., its index section. In other cases it is applied as two component loads acting at the joints at the ends of the lower chord member cut by the section used in determining the load in the member in question. This rule can be used for all chord members and main web members of the truss.

48. The simplest rule for the directions of the elastic weight forces of web members when the chords are not parallel is the same as that for web members of parallel chord trusses—namely, that when the stress in the member indicated positive shear on the truss on a secant action through the member, the moment of the two forces should be counterclockwise. Strictly speaking, the computer does not have just a moment to deal with as the two forces are not equal, but this will cause no difficulty in practice. If an attempt were made to formulate the rule in terms of the direction of the resultant force on the moment center, the rule would become too complex for ordinary use, as it would have to depend on whether the moment center were to the right or the left of the truss.

49. There is an important short cut that can be utilized in computing. It may be noticed that one of the two forces will be equal to ΔL of the member divided by the distance from the member to its secondary moment center—i. e., the far end of the main chord member cut by the index section for the web member in question. In the case of member gf, it is the force $\Delta L/gi$ acting at i. For member dg, it is the force $\Delta L/(eg\cdot\sin\theta)$, acting at e. The other force will be equal to the first plus or minus the numerical value of the elastic weight of the member when considered as a single force. In the case of member gf, this is the force $\Delta L/gi+\Delta L/Og$ acting at g, and for dg it is the force $\Delta L/(eg\cdot\sin\theta)-\Delta L/(Og\cdot\sin\theta)$ acting at g. In all cases, the second of these two forces is the one that acts at the junction of the member in question with the main chord. Whether the term equal to the elastic weight obtained from the chord member rule should be added or subtracted from that obtained from the web member rule is determined by the rule that the numerically larger force is the one nearest to the true moment center of the member.

SECONDARY WEB MEMBERS

50. Many truss designs, such as subdivided Warren trusses, incorporate web members the loads in which are determined by the external forces acting on only a

small part of the span. In general, these members can be handled by the same general method as used in the cases described above—namely, to find the load combination that will produce a moment curve on the phantom beam of the same shape as the influence line for load in the member in question and multiply each of the forces of this loading by ΔL of that member.

51. Member *de* of the truss shown in Figure 1 is of this character. In the first numerical example it caused no difficulty as it was subjected to no load and its ΔL and therefore its elastic weight were obviously equal to zero. If a part of the load acting at *d* had been assumed to have acted at *e* instead, the member *de* would have had to have been considered in the computations. The influence line for load in *de* is a triangle with *cg* as its base and its apex at *e*. The ordinate at *e* will be unity. The force system that will produce a bending moment on the phantom beam numerically equal to the ordinates of this influence line is a single load acting at *e* with reactions at *c* and *g*. The magnitude of the moment at *e* being unity, the reaction at *c* will be $1/ce$. But this reaction must also equal the load at *e* multiplied by eg/cg. Hence the load at *e* will be equal to $\frac{cg}{ce\,.\,eg}$. Multiplying by ΔL we find the elastic weight to be composed of a force at *e* equal to $\frac{cg\,.\,\Delta L}{ce\,.\,eg}$, and reactions of $\Delta L/ce$ at *c*, and $\Delta L/eg$ at *g*. In this, which is the usual case, the secondary web member is vertical and the adjacent panels are equal, so the force at the intersection of that member with the main chord (joint *e*), will be equal to $2\Delta L/cg$, and the forces at *c* and *g* just half as large. It is not considered worth while to attempt to work out the rule for more complex cases.

NUMERICAL EXAMPLE

52. In order to illustrate the application of the rules derived in the above discussion, the deflections of the lower chord joints of the truss shown in Figure 8 have been computed by the method of elastic weights, and the deflection of joint *p* has been checked by the method of work. This truss incorporates practically all of the features that are likely to be found in simply supported trusses encountered in airplane design, though all of them are not likely to be found in any one truss.

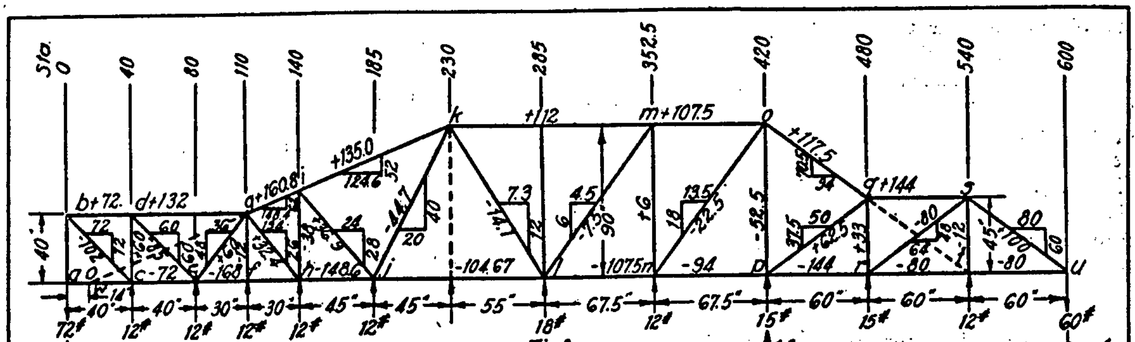

Fig. 8
Simple truss—loads, stresses, and dimensions.

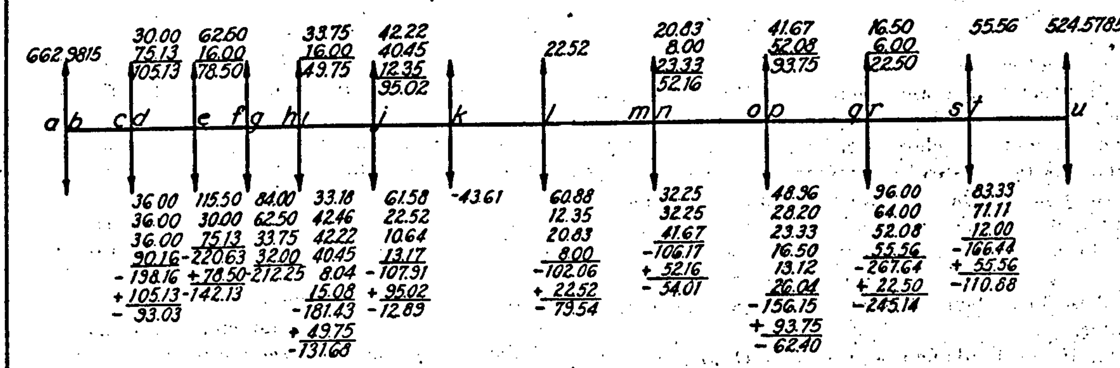

Fig. 9
Elastic weights for truss of fig. 8

53. The computation of the values of *P*, the axial loads in the truss members due to the system of external forces shown in the figure are not given, as they are not difficult and present no phases of special interest.

54. Table 3 gives the complete computations of the elastic weights of the various members of the truss. The values of *P* and *L* were taken from Figure 8. The values of *A* were assumed arbitrarily, the figure selected being in most cases such that the resulting values of $\Delta L = PL/A$ would include no inconvenient fractions. The modulus of elasticity, *E*, was not inserted into the computations for the reason expressed in paragraph 27. The column headed M. C. gives the location of the true moment centers of the members, an asterisk indicating a web member, the index section of which cuts parallel chord members. *H* is the horizontal distance from the moment center to the member. Where a member has both a primary and a secondary

moment center, the horizontal distances from both are given, the distance from the primary moment center being given first and separated from the distance from the secondary moment center by a semicolon. θ is the angle between a member and the horizontal. R is perpendicular distance from the moment center to the member, and r the corresponding distance from the secondary moment center. The column headed S. M. C. shows the locations of the secondary moment centers of the web members; chord members are indicated by asterisks. W_c, the elastic weight as determined by the chord member rule, is equal to $\Delta L/R$. W_w, the elastic weight as determined by the web member rule, is equal to $\Delta L/r$. The last column shows the directions in which the elastic weight forces act. In the case of chord members the indication is by a minus sign for a down and a plus sign for an up load. In the case of the web members a curved arrow shows whether the moment of the forces determined by the web member rule is clockwise or counterclockwise. The column headed P. A. gives the points of application of the elastic weights to the phantom beam.

55. The two secondary web members of the truss of Figure 8, fg and st are both vertical and the adjacent panels are equal. Their elastic weights can therefore be determined by the rule illustrated in paragraph 51. The elastic weight of fg will consist of a downward force of $2\Delta L/ef = 2\times 480/30 = 32$ units acting down at f and forces of 16 units acting up at e and h. The elastic weight of st will be a down force of $2\Delta L/tu = 2\times 360/60 = 12$ units acting down at t and forces of 6 units acting up at r and u. That the forces act in

TABLE 3.—*Computation of elastic weights*

Member	P	L	A	ΔL	M. C.	H.	sin θ	R	S. M. C.	r	W_c	W_w	Sign	P. A.
bd	+72	40	2.00	1,440	c			40	•		36.00		−	c
dg	+132	70	2.0	4,620	e			40	•		115.50		−	e
gi	+160.8	32.5	3.25	1,608	h	126	5/13	48.46	•		33.18		−	h
ik	+135	97.5	3.25	4,050	j	171	5/13	65.77	•		61.58		−	j
km	+112	122.5	2.5	5,488	l			90	•		60.98		−	l
mo	+107.5	67.5	2.5	2,902.5	n			90	•		32.25		−	n
oq	+117.5	75	2.5	3,525	p	120	− 3/5	72	•		48.96		−	p
qs	+144	60	2.0	4,320	r			45	•		96.00		−	r
su	+100	75	2.5	3,000	t	60	3/5	36	•		83.33		−	t
ac	0	40	2.0	0	b			40	•				−	b
ce	−72	40	2.0	1,440	d			40	•		36.00		−	d
efh	−168	60	3.0	3,360	g			40	•		84.00		−	g
hj	−148.6	45	3.0	2,229	i			52.5	•		42.46		−	i
jl	−104.7	100	2.67	3,925	k			90	•		43.61		−	k
ln	−107.5	67.5	2.50	2,902.5	m			90	•		32.25		−	m
np	−94	67.5	2.5	2,538	o			90	•		28.20		−	o
pr	−144	60	3.0	2,880	q			45	•		64.00		−	q
rtu	−80	120	3.0	3,200	s			45	•		71.11		−	s
ab	+72	40	2.0	1,440	•	40	1.0		e	40		36.00	↷	ac
bc	−102	56.5	2.26	2,550	•	40	.707		a	28.28		90.16	↷	ac
cd	+60	40	2.0	1,200	•	40	1.0		e	40		30.00	↷	ce
de	−85	56.5	2.26	2,125	•	40	.707		e	28.28		75.13	↷	ce
eg	+60	50	2.0	1,500	•	30	.8		f	24		62.50	↷	ef
fg	−12	40	1.0	480	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	efh
gh	+32.4	50	2.0	810	w	126; 30	.8	100.8	f	24	8.04	33.75	↶	fh
hi	−38	52.5	1.05	1,900	w	126; 45	1.0	126	j	45	15.08	42.22	↶	hj
ij	+36.9	69.15	1.845	1,382	w	171; 45	52.5/69.15	129.83	h	34.17	10.64	40.45	↶	hj
jk	−44.7	100.62	2.235	2,014	w	171; 100	90/100.62	152.95	l	89.45	13.17	22.52	↶	jl
kl	−14.1	105.47	1.41	1,054	•	100	90/105.47		j	86.33		12.35	↷	jl
lm	−7.5	112.5	0.75	1,125	•	67; 5	.8		n	54		20.83	↶	ln
mn	+6	90	1.0	540	•	67; 5	1.0		l	67.5		8.00	↶	ln
no	−22.5	112.5	1.125	2,250	•	67; 5	.8		p	54		41.67	↶	np
op	−52.5	90	3.0	1,575	t	120; 67.5	1.0	120	n	67.5	13.12	23.33	↷	np
pq	+62.5	75	2.5	1,875	t	120; 60	.6	72	r	36	26.04	52.08	↷	pr
qr	+33	45	1.5	990	•	60	1.0		p	60		16.50	↶	pr
rs	−80	75	3.0	2,000	•	60	.6		•	36		55.56	↶	rt
st	−12	45	1.5	360	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	rtu

[1] See par. 55.

the directions stated can be seen from the fact that the deflection of points f and t will be greater than if the external loads at those points had been applied at g and s, and the elastic weights would have to act in the directions stated in order to bring about that result in the deflection computations.

56. The computations of the elastic weight of fg can be checked by the method of work as follows: If we were to compute the deflection of point f by the latter method, we would apply an upward unit load at f which would subject member fg to unit compression. From the formula of paragraph 3, that part of the deflection of f due to the deformation of fg would be equal to $\Delta L.s$. As ΔL in this case is -480, and s is -1, the deflection in question would be $+480$ units. The minus sign for ΔL is due to the fact that its axial load as shown in Figure 8 is compression. Returning to the elastic weight computations, that part of the deflection of f due to deformation of fg will be numerically equal to the bending moment on the phantom beam at f due to the elastic weight of fg. This will be plus $16 . 30 = 480$ units, checking the computation by the method of work. This method of check is simple and will be found to be of great assistance in determining the elastic weights of members in doubtful cases.

57. The phantom beam with the elastic weights taken from Table 3 and paragraph 55 indicated is shown in Figure 9, along with the net elastic loads at each joint and the reactions at a and u. The computations of these reactions are simple and are not shown. Table 4 shows the computations of the moments on the phantom beam which are numerically the deflections of the lower chord joints of the truss. The error of one unit is negligible and is obviously due to lack of more significant figures in the computations.

58. The correctness of the elastic weight computations were checked by computing the deflection of

joint p by the method of work. The computations are shows in Table 5. The values of ΔL were taken from Table 3, the signs indicating whether the load in the member is tension or compression. The column headed u_p gives the magnitudes of the loads in the various members due to a unit upward load at joint p. As shown in the table, the discrepancy between the values for the deflection of joint p, obtained by the two methods is only 1 in 4,150, and it is reasonable to assume that it is due entirely to the lack of sufficient significant figures in the computations.

TABLE 4.—*Moments on phantom beam*

Station	Shear	ΔL	$S\Delta L$	M
ab				0
	662.9815	40	26,519	
cd				26,519
	569.9515	40	22,798	
e				49,317
	427.8215	30	12,835	
fg				62,152
	215.5715	30	6,467	
hi				68,619
	83.8915	45	3,775	
j				72,394
	71.0015	45	3,195	
k				75,589
	27.3915	55	1,507	
l				77,096
	−52.1485	67.5	−3,520	
mn				73,576
	−106 1585	67.5	−7,166	
op				66,410
	−168.5585	60	−10,114	
qr				56,296
	413.6985	60	−24,822	
st				31,474
	−524.5785	60	−31,475	
u				−1 error.

TABLE 5.—*Check by method of work deflection at joint P*

Member	ΔL	u_p	$u_p\Delta L$
bd	+1,440	+0.300	+432
dg	+4,620	+.600	+2,772
gi	+1,608	+.866	+1,393
ik	+4,050	+.845	+3,422
km	+5,488	+.950	+5,214
mo	+2,902.5	+1.175	+3,410
oq	+3,525	+1.750	+6,169
qs	+4,320	+1.867	+8.065
su	+3,000	+1.167	+3,500
ac	−0	0	0
ce	−1,440	−.300	+432
efh	−3,360	−.825	+2,772
hj	−2,229	−.800	+1,783
jl	−3,925	−.767	+3,010
ln	−2,902.5	−1.175	+3,410
np	−2,538	−1.400	+3,553
pr	−2,880	−1.867	+5,377
rtu	−3,200	−.933	+2,986
ab	+1,440	+.300	+432
bc	−2.550	−.425	+1,084
cd	+1,200	+.300	+360
de	−2,125	−.425	+903
eg	+1,500	+.375	+563
fg	−480	0	0
gh	+810	+.042	+34
hi	−1,900	−.033	+63
ij	+1,382	+.032	+44
jk	−2,014	−.027	+54
kl	−1,054	−.352	+371
lm	−1,125	+.375	−422
mn	+540	−.300	−162
no	−2,250	+.375	−844
op	−1,575	−1.350	+2,126
pq	+1,875	+.583	+1,093
qr	+990	+.700	+693
rs	−2,000	−1.167	+2,334
st	−360	0	0
By method of work Eδ			+66,426
By elastic weights Eδ			+66,410
Error			16

Error=16 in 66,400=1 in 4,150 due to lack of significant figures.

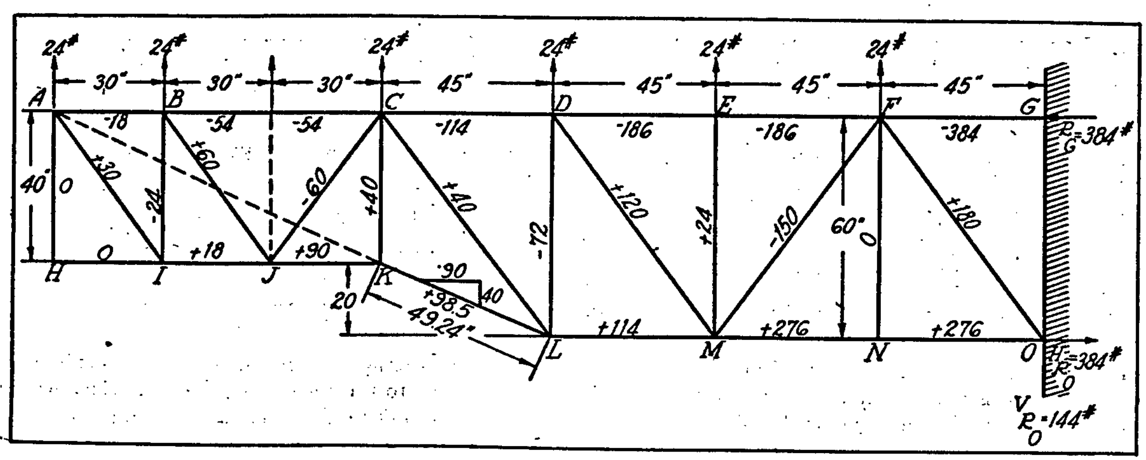

FIG. 10.—Cantilever truss loads and dimensions

CANTILEVER TRUSSES

59. Cantilever trusses can be treated by the same rules as those resting on two supports, except that the phantom beam, instead of being supported at the same locations as the truss, must be assumed to be supported at the free end of the actual truss, the free end of the phantom beam being at the same location as the fixed end of the actual truss. The rules for the magnitude of the elastic weights and for their directions remain the same. The detailed proof of this statement will not be gone into in this report, but the reader who wishes to understand the method more thoroughly will find the checking of the statement an excellent means for so doing. The application of the method to a cantilever truss is illustrated in the following paragraphs, the truss and loading used being shown in Figure 10. To show the validity of the method in this case, the deflection of the tip of the cantilever is also worked out by the method of work. In this example, instead of considering the lower as the main chord and computing the deflections of the joints of the lower chord, the deflections of the upper chord are computed.

60. The truss with the external loads and resulting axial loads in the members is shown in Figure 10. The elastic weight computations are given in Table 6, in which the arrangement and nomenclature are the same as in Table 3. Figure 11 shows the phantom beam with the loads on it, and also the computations of the deflection of the tip A. H. The deflections of the other joints of the upper chord could be very easily obtained by computing the curve of moments on the phantom beam from the loads shown. Table 7 gives a check computation of the deflection of point A by the method of work, but presents no features of special interest.

TABLE 6.—*Computation of elastic weights, cantilever truss*

Member	P	L	A	$ALE=\frac{PL}{A}$	M. C.	H	Sin θ	R	S. M. C.	r	W_c	W_w	Sign	P. A.
AB	−18	30	1.8	300	I			40	•		7.50		+	I
BC	−54	60	1.8	1,800	J			40	•		45.00		+	J
CD	−114	45	1.9	2,700	L			60	•		45.00		+	L
DE	−186	45	2.0	4,185	M			60	•		69.75		+	M
EF	−186	45	2.0	4,185	M			60	•		69.75		+	M
FG	−384	45	3.0	5,760	O			60	•		96.00		+	O
HI	0	30	2.0	0	A			40	•				+	A
IJ	+18	30	1.8	300	B			40	•		7.50		+	B
JK	+90	30	1.8	1,500	C			40	•		37.50		+	C
KL	+98.5	49.24	2.0	2,425	C	90	0.4062	36.56	•		66.33		+	C
LM	+114	45	1.9	2,700	D			60	•		45.00		+	D
MN	+276	45	3.0	4,140	F			60	•		69.00		+	F
NO	+276	45	3.0	4,140	F			60	•		69.00		+	F
AH	0	40	2.0	0	•	•			•	•				
AI	+30	50	2.0	750	•	30	.8		B	24		31.25	↶	AB
BI	−24	40	1.0	960	•	30	1.0		A	30		32.00	↶	AB
BJ	+60	50	1.0	3,000	•	60	.8		C	48		62.50	↶	BC
JC	−60	50	1.0	3,000	•	60	.8		B	48		62.50	↶	BC
CK	+40	40	1.0	1,600	[1] C	[1] 90	[1] .4062	36.56	•		17.78		[1]	[1] C
CL	+40	75	2.0	1,500	A	90, 45	.8	72	D	36	20.83	41.67	↶	CD
DL	−72	60	2.0	2,160	•	45	1.0		C	45		48.00	↶	CD
DM	+120	75	3.0	3,000	•	45	.8		E	36		83.33	↶	DE
EM	+24	60	1.0	1,440	•	45	1.0		D or F	45		[2]	[2]	[2]
MF	−150	75	3.0	3,750	•	45	.8		E	36		104.17	↶	EF
FN	0	60		0	•		1.0		•	•				
FO	+180	75	3.0	4,500	•	45	.8		G	36		125.00	↶	FG

[1] For treatment of member, CK see par. 61.
[2] Elastic weight of EM consists of 64 units acting down at E and 32 units at D and F acting up at both joints. (See par. 51.)

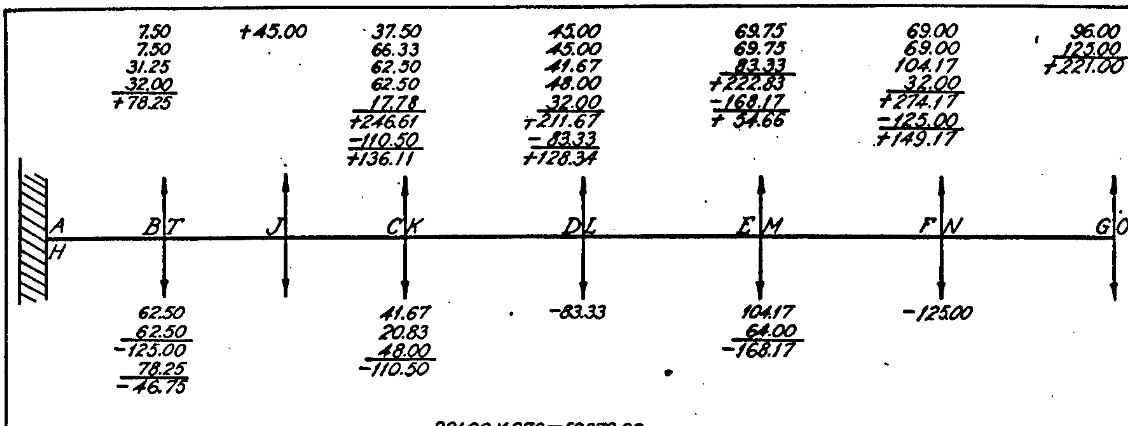

Fig. 11. Phantom beam of truss of fig. 10 with deflection computations.

Table 7.—*Computation of deflection of joint A by method of work*

Member	ΔL	u_A	$u_A \Delta L$
AB	−300	−0.75	+225
BC	−1,800	−1.50	+2,700
CD	−2,700	−2.25	+6,075
DE	−4,185	−3.00	+12,555
EF	−4,185	−3.00	+12,555
FG	−5,760	−4.50	+25,920
HI	0	0	
IJ	+300	+0.75	+225
JK	+1,500	+2.25	+3,375
KL	+2,425	+2.462	+5,970.35
LM	+2,700	+2.25	+6,075
MN	+4,140	+3.75	+15,525
NO	+4,140	+3.75	+15,525
AH	0	0	0
AI	+750	+1.25	+937.5
BI	−960	−1.00	+960
BJ	+3,000	+1.25	+3,750
JC	−3,000	−1.25	+3,750
CK	+1,600	+1.00	+1,600
CL	+1,500	0	0
DL	−2,160	−1.00	+2,160
DM	+3,000	+1.25	+3,750
EM	+1,440	0	0
MF	−3,750	−1.25	+4,687.5
FN	0	0	0
FO	+4,500	+1.25	+5,625
Deflection of joint A, Eδ			133,945.35
By elastic weights, Eδ			133,945.35
Error			0

61. The only member of the truss of Figure 10, which should cause difficulty is CK and special methods are required for it. There is no section through the truss which can be used directly for computing the load in this member by the method of moments, so that it has no true moment center, and the rule for chord members can not be applied to it. Furthermore, it has no secondary moment center like the normal web members, so that the web member rule is equally inapplicable. The load in this member is found from the fact that it is numerically equal to and of the same sign as vertical component of the load in KL, and this makes it possible to determine its elastic weight. If we designate the ratio of the load in CK to the load in KL by k, we can say that for the application of the basic formula in paragraph 3 that s_{CK} equals ks_{KL}. As the deflection of any point on the truss due to deformation of CK is equal to $\Delta L_{CK}.s_{CK}$, it will be also equal to $\Delta L_{CK}.ks_{KL}$. From this it can be shown that the elastic weight of CK can be obtained by treating it as if it were in the location of KL and multiplying the result by k. This was done in Table 6 where C is given as the moment center of CK, and R as 36.56, the value for KL. The value of the elastic weight, however, is given as $\frac{1{,}600 \times 0.4602}{36.56} = 17.78$, the quantity 0.4602 being the ratio k.

62. The computations for the elastic weight of CK can be checked by the method of moments as follows: From Table 7, the deflection of point A due to deformation of CK is 1,600 units. By the elastic weight computation, using 17.78 pounds acting up at C as the elastic weight of that member, the same quantity would be $17.78 \times 90 = 1{,}600.2$ units, the difference being due to the fact that only four significant figures were retained in the computation of the figure 17.78.

63. It may be noted that the check of the deflection of point A by the method of work does not throw any light on whether the elastic weight of member EM was computed properly, as the elastic weight of this member is composed of three forces that are in equilibrium and do not affect the bending moments on the phantom beam outside of the section DF, and the same would be true of any three forces at these points which were also in equilibrium. In fact the computation of the elastic weight of EM could be neglected if the deflection of point E were not desired. The only way to determine whether or not

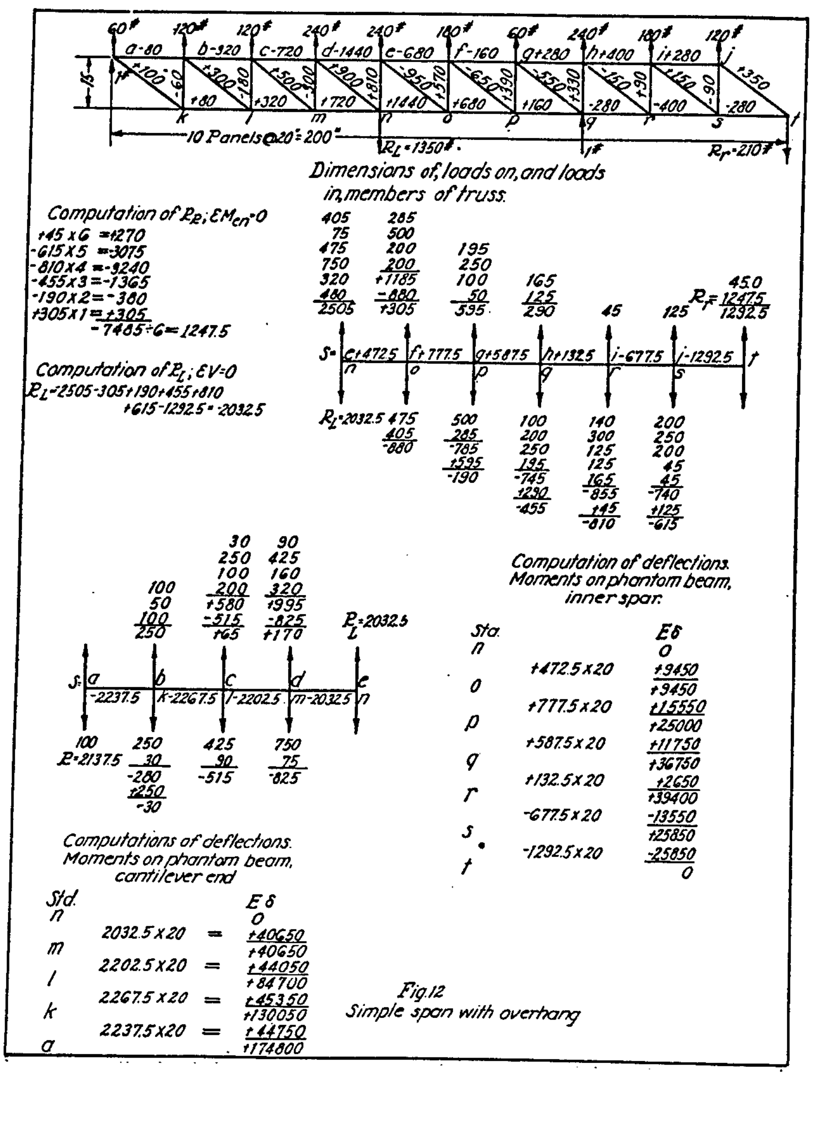

Fig. 12
Simple span with overhang

this elastic weight was computed correctly is to check it in the manner employed for member fg of Figure 8. This will not be done here, but would be a good exercise for the reader.

SIMPLE SPAN WITH OVERHANG

64. In the example of a cantilever truss given above, it was assumed that the truss was fixed at one end, the tangent to the elastic curve of the truss at the fixed end being horizontal. In airplane structures, a more common case is that of a simple span with a pin joint at one end and a cantilever at the other. For this type of truss the procedure is as follows: First, the elastic weights of all members are computed and applied to the phantom beam. Second, the phantom beam of the portion of the truss between the supports is treated as though the cantilever were absent, the reactions at the supports and the bending moments on it being computed. The phantom beam corresponding to the cantilever portion of the truss is then considered, it being treated as a cantilever fixed at the free end of the actual truss. In this step a force equal in magnitude but opposite in direction to the reaction on the phantom beam of the simply supported span at the root of the cantilever is treated as a load on the phantom beam of the cantilever portion.

65. The computer may have some question as to how to handle those elastic weight forces that act at the root of the cantilever, but a little consideration will show that they should give no trouble. If such a force is assumed to act on the phantom beam of the simply supported span, the reaction at its point of application will include an equal and opposite force, so it will have no effect on the computation of the deflections between the two supports. A force equal but opposite to the reaction on the phantom beam of the simply supported span is applied to the phantom of the cantilever at the junction of the two portions. The net load on the phantom beam of the cantilever will therefore be the same whether the elastic loads at the point in question are first considered to act on the simply supported span, and then to be applied to the phantom beam of the cantilever as part of the force equal and opposite to the reaction, or are assumed to act directly on the phantom beam of the cantilever portion of the truss. The chief point to remember is that such elastic weight forces must be considered once and only once, but whether they are assumed to act on the phantom beam of the cantilever or that of the simply supported span of the truss makes no difference in the final results.

66. A numerical example of the computations for this case is given below for the truss shown in Figure 12. The example includes computations of the deflections of the cantilever tip a and a joint q in the simply supported span.

TABLE 8.—*Computation of deflection simple span with overhang—Computation of elastic weights*

Member	P	L	A	ΔL	E. W.	Sign	P. A.	
							Up	Down
ab	−80	20	16/15	−1,500	100	+	bk	
bc	−320	20	32/15	−3,000	200	+	cl	
cd	−720	20	45/15	−4,800	320	+	dm	
de	−1,440	20	60/15	−7,200	480	+	en	
ef	−680	20	68/15	−3,000	200	+	fo	
fg	−160	20	64/15	−750	50	+	gp	
gh	+280	20	56/15	+1,500	100	−	------	hq
hi	+400	20	80/21	+2,100	140	−	------	ir
ij	+280	20	28/15	+3,000	200	−	------	js
jt	+350	25	35/12	+3,000	250	−	------	js
kl	+80	20	32/15	+750	50	+	bk	
lm	+320	20	64/15	+1,500	100	+	cl	
mn	+720	20	90/15	+2,400	160	+	dm	
no	+1,440	20	90/15	+4,800	320	+	en	
op	+680	20	68/15	+3,000	200	+	fo	
pq	+160	20	32/15	+1,500	100	+	gp	
qr	−280	20	28/15	−3,000	200	−	------	kq
rs	−400	20	16/9	−4,500	300	−	------	ir
st	−280	20	28/15	−3,000	200	−	------	js
ak	+100	25	25/12	+1,200	100	↶	bk	a
bl	+300	25	25/10	+3,000	250	↶	cl	bk
cm	+500	25	25/51	+5,100	425	↶	dm	cl
dn	+900	25	25/10	+9,000	750	↶	en	dm
eo	−950	25	25/6	−5,700	475	↷	en	fo
fp	−650	25	65/24	−6,000	500	↷	fo	gp
gq	−550	25	55/12	−3,000	250	↷	gp	hq
hr	−150	25	25/10	−1,500	125	↷	hq	ir
is	+150	25	25/10	+1,500	125	↶	js	ir
bk	−60	15	1.5	−600	30	↶	cl	bk
cl	−180	15	1.5	−1,800	90	↶	dm	cl
dm	−300	15	3.0	−1,500	75	↶	en	dm
en	+810	15	1.5	+8,100	405	↷	en	fo
fo	+570	15	1.5	+5,700	285	↷	fo	gp
gp	+390	15	1.5	+3,900	195	↷	gp	hq
hq	+330	15	1.5	+3,300	165	↷	hq	ir
ir	+90	15	1.5	+900	45	↷	ir	js
js	−90	15	1.5	−900	45	↶	t	js

TABLE 9.—*Computation of deflections of joints a and q by method of work*

Member	ΔL	u_q	$\Delta L u_q$	u_a	$\Delta L u_a$
ab	−1,500	0	0	−4/3	+2,000
bc	−3,000	0	0	−8/3	+8,000
cd	−4,800	0	0	−4	+19,200
de	−7,200	0	0	−16/3	+38,400
ef	−3,000	+2/3	−2,000	−40/9	+13,333 1/3
fg	−750	+4/3	−1,000	−32/9	+2,666 2/3
gh	+1,500	+2.0	+3,000	−8/3	−4,000
hi	+2,100	+4/3	+2,800	−16/9	−3,733 1/3
ij	+3,000	+2/3	+2,000	−8/9	−2,666 2/3
jt	+3,000	+0.8 1/3	+2,500	−10/9	−3,333 1/3
kl	+750	0	0	+4/3	+1,000
lm	+1,500	0	0	+8/3	+4,000
mn	+2,400	0	0	+4	+9,600
no	+4,800	0	0	+16/3	+25,600
op	+3,000	−2/3	−2,000	+40/9	+13,333 1/3
pq	+1,500	−4/3	−2,000	+32/9	+5,333 1/3
qr	−3,000	−2.0	+6,000	+8/3	−8,000
rs	−4,500	−4/3	+6,000	+16/9	−8,000
st	−3,000	−2/3	+2,000	+8/9	−2,666 2/3
ak	+1,200	0	0	+5/3	+2,000
bl	+3,000	0	0	+5/3	+5,000
cm	+5,100	0	0	+5/3	+8,500
dn	+9,000	0	0	+5/3	+15,000
eo	−5,700	−5/6	+4,750	−10/9	+6,333 1/3
fp	−6,000	−5/6	+5,000	−10/9	+6,666 2/3
gq	−3,000	−5/6	+2,500	−10/9	+3,333 1/3
hr	−1,500	+5/6	−1,250	−10/9	+1,666 2/3
is	+1,500	+5/6	+1,250	−10/9	−1,666 2/3
bk	−600	0	0	−1	+600
cl	−1,800	0	0	−1	+1,800
dm	−1,500	0	0	−1	+1,500
en	+8,100	+0.5	+4,050	+2/3	+5,400
fo	+5,700	+0.5	+2,850	+2/3	+3,800
gp	+3,900	+0.5	+1,950	+2/3	+2,600
hq	+3,300	−0.5	−1,650	+2/3	+2,200
ir	+900	−0.5	−450	+2/3	+600
js	−900	−0.5	+450	+2/3	−600
	--------	--------	36,750	--------	+174,800

CONCLUSION

67. It is the belief of the writer that the method of elastic weights described above provides a simple and workable method of computing the deflections of long shallow trusses of many panels. It is probable that it can also be used to develop methods by which "effective moments of inertia" can be derived for the more common cases, these effective moments of inertia to be used in computing truss deflections by means of the formulas for the deflection of beams, but no work along these lines has been started as yet. It is hoped that this line of study can be followed in the near future.

68. In the introduction to this report the rules to be used in applying the method of elastic weights were stated. In the text, some of the special cases of these rules were proved, but no attempt was made to give a rigid proof of them as general propositions. This could have been done, but would have involved a great increase in the length of the report, and would have greatly delayed its publication. The writer has tested them for all normal cases likely to be met in airplane work and believes them to be true generally. The discussions in the report were aimed not so much to give rigid proofs of the rules, but to indicate how they were obtained, indicate to those unfamiliar with the method, how report or rules can be used, and demonstrate to him their reasonableness.

APPENDIX

RULES AND DEFINITIONS

1. In the body of the report, the method of computing truss deflections was described and illustrated by numerical examples. In the course of this discussion numerous rules were formulated, and the meanings of several special terms were defined, but these rules and definitions are scattered through the text in such a manner that they would be difficult to find by a person desiring to use this report as a guide in computing the deflections of a truss. Furthermore, many of the rules are given in the form in which they apply to the special case being discussed, the expansion of them to fit the other cases being described rather than defined. For these two reasons, it has been considered advisable to restate the rules in more general terms, and to place these rules and the accompanying definitions in this appendix for ready reference.

2. *The method of elastic weights* is fundamentally based on the same principles as the more familiar *method of work*, the chief difference being that some mathematical short cuts are used to facilitate the computation of the deflections of all of the panel points on one chord of the truss. This fact should be kept in mind and utilized in doubtful cases by checking the deflections of some point due to the deformation of the member causing trouble by the two methods. If the results of the two methods check, the computer can feel satisfied that he has computed the elastic weights of those members correctly. Examples of such checks are given in paragraphs 56 and 62.

3. In order to compute the deflection of a truss by the method of elastic weights, a force or moment, called the *elastic weight*, is computed for each member of the truss. The curve of bending moments on a *phantom beam* due to these elastic weights is numerically equivalent to the curve of deflections of the *main chord* of the truss under the loading investigated.

4. When the deflection of a truss is computed, as a general rule, only the deflections of the joints of one chord are desired. In the following discussion it will be assumed that this is the case and the chord, the deflections of which are being computed, will be called the *main chord* of the truss. Whether the main chord in any given case is the upper chord or the lower depends upon the special conditions of the problem. If the deflections of the joints of both chords are desired, two sets of computations must be made, though much of the two sets will be identical. In practice this condition will seldom arise, and in the remainder of this report it will be assumed that the deflections of only one chord are desired.

5. The *phantom beam* is a hypothetical beam assumed to be subjected to the elastic weights of the truss members. In the case of simply supported truss spans it is assumed to be supported at the same points as the truss span. In the case of cantilever trusses the phantom beam is assumed to be supported at the free end of the actual truss, the end of the phantom beam at the point of support of the actual truss being assumed to be a free end of a cantilever.

6. When the deflections are to be computed for a simple span with an overhang, the elastic weights of the members in the inner span are computed and applied to its phantom beam. The reactions on the phantom beam at the points of support of the truss and the bending moments on the phantom beam of the span are then computed, these moments being numerically equal to the deflections of the inner span of the truss. In computing the deflections of the cantilever overhang, the outer reaction of the phantom beam for the inner span is applied to the phantom beam for the cantilever portion, in addition to the elastic weights of the members of the cantilever portion of the truss. The phantom beam of the cantilever is considered as a cantilever supported at the free end of the actual truss. The curve of moments on this beam gives the deflection of the cantilever portion of the truss measured from the line joining the points of support at the ends of the inner span. A numerical example of this case is given in paragraphs 10 to 29.

7. The *elastic weight* of a truss member is a force, acting at the *moment center* of the member, numerically equal to $\Delta L_1 = PL/AE$, the change in length of the member under the system of external loads on the truss, divided by the *arm* of the member about its moment center. In this report this rule is called the *chord member rule* for determining the elastic weight.

8. The *moment center* of a member is the intersection of the other two members cut by the section used in determining the load in that member by the method of moments. In this report this section is called the *index section* of the truss for the member.

9. In the cases of web members of parallel chord trusses, the moment centers are at an infinite distance from the members, and the above rule for the determination of the elastic weight can not be applied. The elastic weight of such a member can be considered as either a moment numerically equal to ΔL, or as two equal and opposite forces, each equal to ΔL divided by the arm of the member about its *secondary moment center*, one acting through the secondary moment center and the other at the intersection of the member and the main chord of the truss. In this report this rule is called the *web member rule* for determining the elastic weight.

10. The *secondary moment center* of a member is the distant end of the main chord member cut by the index section for the web member in question.

11. When the moment center of a member is at one end of a chord member cut by its index section, the elastic weight of that member is assumed to be applied to the phantom beam at that moment center. This is the normal case for chord members.

12. When the moment center of a member is not at one end of a chord member cut by its index section, the elastic weight is applied to the phantom beam as two forces, one at the secondary moment center of the member, and the other at the intersection of the member with the main chord of the truss. In the case of a web member the index section of which cuts parallel chord members, each of these forces will be equal to ΔL divided by the arm of the member about its secondary moment center. In other words, the elastic weight can be determined at once from the web member rule. In the case of a web member the index section of which cuts nonparallel chords, the two forces will be of such magnitude that their resultant will be equal to ΔL, divided by the arm of the member about its moment center, acting through the moment center. In other words, the elastic weight will be composed of two forces, the resultant of which can be determined by the chord member rule.

13. In the case of web members between nonparallel chords, the computation of the elastic weight forces applied to the phantom beam can be most easily accomplished as follows. First apply a couple composed of two forces, each equal to ΔL divided by the arm of the member about its secondary moment center, at the secondary moment center and the intersection of the web member and the main chord. In short, first apply the elastic weight as obtained from the web member rule to the phantom beam. Then apply a force equal to ΔL divided by the arm of the member about its moment center—that is, a force of the magnitude of the elastic weight as obtained from the chord member rule at the intersection of the member and the main chord. This last force should act in such a direction that the numerically larger of the two forces constituting the elastic weight of the member will be the one nearest to its moment center. These two forces will conform to the criterion of the preceding paragraph.

14. The sign indicating the direction of the elastic weight of a chord member shall be the same as the sign of the bending moment on the truss at the index section of that member as indicated by the character of the load in the chord member. Thus the elastic weight of an upper chord member in compression or of a lower chord member in tension will be a positive or upward acting force; while the elastic weight of an upper chord member in tension or a lower chord member in compression will be a negative or downward acting force on the phantom beam.

15. If the shear on the index section of a web member, as indicated by the character of the stress in that member is positive, the left-hand force of its elastic weight shall act down and the right-hand force up—i. e. the forces shall produce a counterclockwise couple on the phantom beam. If this shear is negative, the forces of the elastic weight should act in the opposite directions and form a clockwise couple. In this rule, the phrase "as indicated by the character of the stress in that member" is important as in cases where the index section cuts nonparallel chords, the character of the shear on the truss as indicated by the stress in the web member may differ from the true shear on the section. Member *ij* in Figure 8 and some of the adjacent web members illustrate cases of this kind.

16. *Special members.*—The rules stated above will not be found applicable to all members of some truss designs. The most common example of members of this character are the subverticals of a subdivided Warren truss. No general rule can be formulated to take care of all possibilities of this kind, but the body of the report gives examples of how to handle the more common cases. A general rule that can be used to check the correctness of the application of the method of elastic weights in doubtful cases is that this method and that of work should give the same results. It will nearly always be a simple matter to make a check of this character.

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